Residual stress is more tractable than residual strain in models of the heart and vessels

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The residual stress in the heart and vessels is fundamental to their mechanical behavior and their growth and remodeling. Yet, residual stress and strain are often neglected to make problems more tractable or solvable. One possible reason for this situation is that the cardiovascular community attempts to measure and model residual strain rather than residual stress. Residual strain is inherently difficult to define because it must violate compatibility—to relieve residual strain one needs to make at least one cut. Moreover, recent experiments on rings of hearts [1] and vessels [2] show that many cuts are needed. In particular, a circumferential cut after a radial cut will relieve more strain. Although often used as a model of the residual strain, the closing of a bent arc clearly violates this observation of the residual strain. The residual stress field, nevertheless, must satisfy static equilibrium.

In essence, residual stress is potentially more tractable than residual strain because residual stress must satisfy static equilibrium whereas residual strain must violate compatibility. Residual stress, moreover, is easy to introduce into a model where the reference configuration is an intact load-free one. This is so because the linear terms in the strain-energy function (per unit reference volume) W are the stress components in the reference configuration. To model experiments on rings of heart tissue (wherein a circumferential cut was made after a radial cut), we introduced linear terms into W that varied with radius. Since the residual stress must satisfy equilibrium, the residual radial stress was automatically determined once the residual hoop stress was given. A stress-free configuration did not need to be defined. In-fact, there would be an infinite number of discontinuities (i.e., cuts) in the residual strain field for our simple, well-behaved model.

References

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